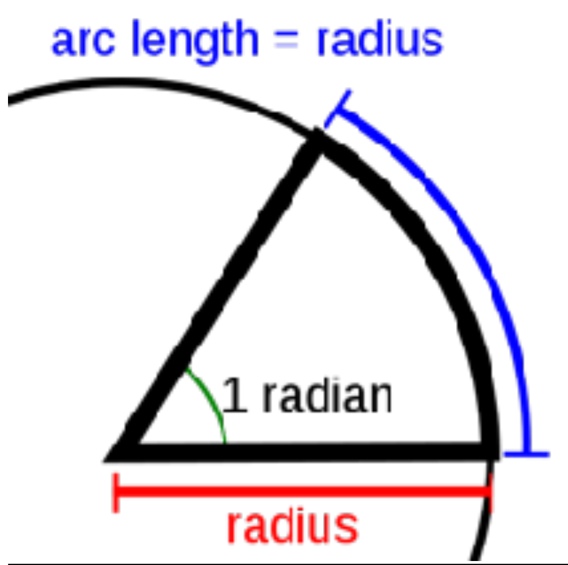
What is a Mil (Mil Dot Reticle) and does it work as a "hold over" aiming method?

Ward W Brien 2012©

Does a milliradian reticle design work using the hold-over method? Yes and No. But first, let's review some "milliradian" facts.

Definition

Radian describes the plane angle subtended by a circular arc the length of the arc divided by the radius of the arc. One radian is the angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle. More generally, the magnitude in radians of such a subtended angle is equal to the ratio of the arc length to the radius of the circle; that is, $\underline{\theta} = s/r$, where θ is the subtended angle in radians, s is arc length, and r is radius. Conversely, the length of the enclosed arc is equal to the radius multiplied by the magnitude of the angle in radians; that is, $s = r\theta$. <u>In other words, if the radius of a circle equals an outside portion of the arc of the circle devided by 1,000, it then equals one radian. A Milliradian is a radian divided by 1,000.</u>



O.K., so as a manufacturer of rifle scopes, or a supplier to our Troops, or an Instructor who is training our Troops, you theoretically posses a crystal clear understand of what a milliradian is… However, not to leave anything to a chance misunderstanding, I will cover the history of radian measure.

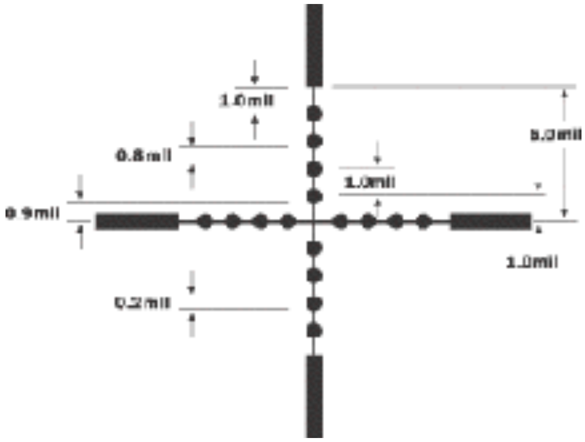
History

The concept of radian measure, as opposed to the degree of an angle, is normally credited to Roger Cotes in 1714. He had the radian in everything but name and he recognized its naturalness as a unit of angular measure. The idea of measuring angles by the length of the arc was used already by other mathematicians. For example al-Kashi (c. 1400) used so-called ***diameter parts*** as units where one diameter part was 1/60 radian and they also used sexagesimal subunits of the diameter part.

The term ***radian*** first appeared in print on 5 June 1873, in examination questions set by James Thomson (brother of Lord Kelvin) at Queens College, Belfast. He used the term as early as 1871, while in 1869, Thomas Muir, then of the University of St Andrews, vacillated between ***rad*, *radial*** and ***radian***. In 1874, Muir adopted ***radian*** after a consultation with James Thomson.

A well known Rifle Scope manufacturer's definition of the Milliradian reticle and its use is as follows:

"Here's a brief history on the military mil and its comparison to the milliradian. Sometime prior to WWI with the advent of precision artillery, the military decided to come up with a precision compass unit. The milliradian was in the ballpark of what they were looking for, but 6283.19 milliradians to 360 degrees would have made the math difficult. So the military shrank the milliradian by about 2%, and wound up with 6400 mills to 360 degrees. Why 6400 versus a simple rounding to 6300??? Well 6400 is easily divisible by 8, which corresponds to the primary cardinal directions (i.e. N, NE, E, SE, S, SW, W, NW) and their subdivisions. So (as far as I know), that is how the military "mil" was created. The mil dot reticles that we produce are based on the milliradian. The reason we do that, is that it fulfills the 1000 to 1 ranging ratio which the military wanted. What this means is that 1 milliradian will subtend a 1 meter target at 1000 meters (or a 1 yard target at 1000 yards, a 1 foot target at 1000 feet.....you get the picture). The milliradian does this exactly, thus it was chosen. Now when we compare the military "compass mil" and the milliradian, they are rather close: 1.02 military mills (3.375 moa) = 1.00 milliradian (3.439 moa). As you can see the difference is miniscule.....it roughly corresponds to a 2 centimeter difference on a 1 meter target at 1000 meters, or a 2 millimeter difference on a 1 meter target at 100 meters. That's a 0.079"!!!! So even with a 1/4 moa barrel and 1/4 moa adjustments on the scope itself, it would make no difference to the shooter whether he calculates the distance using the milliradian or the mil. As far as ranging is concerned, the difference is similar: using the military mil, a 1 meter target at 1000 meters would be ranged at 980 meters. At 100 meters, the 1 meter target would be ranged at 98 meters. I seriously doubt whether anyone can actually use a mil dot reticle to that degree of accuracy anyway. In practicality, most modern military cartridges do not drop like a rock. If one is shooting out to 1000 meters, they are using a 300 WM or a 338 Lapua, which will not have a significant enough drop in the 1000 meter ballpark to reflect a 20 meter difference. So, as you can see the difference between the two is rather insignificant to all but a few world class bench rest shooters (if that).



Mil Dots as aiming points

Utilizing Mil Dots as aiming points, which is the "Dots" of the mil dot system, requires knowing which Dot to use for each 50/100 yd increment for the entire trajectory of your bullet. The Dots designated for long range will have to be the aiming point for a series of yardage increments. The amount of hold from target center will be different for each increment depending on the distance. You may have to hold the designated Dot low from target center for one 50 yd increment, then high for the next 50 yds. There is no consistent pattern to go by. Each high and low hold from target center will range anywhere from several inches to a few feet depending on the distance. For some long-range shots, you will have to place the appropriate Dot literally above or below your target for the proper bullet drop compensation. This provides no real aiming point to focus on which is a crucial factor for accurate long range shooting. The disadvantages of utilizing Mil Dots as aiming points for bullet drop compensation are a s follows: The limited number of Mil Dots having to be utilized as aiming points for so many yardage increments creates the problem of so many different holds on your target. Shooting at high altitudes or extreme temperatures requires different holds than that applied for the field conditions at your home range. The size of a Dot covers up too much of your target for a precise shot at long, as well as, medium ranges. The dot completely covers up small or partially concealed targets at medium to long-range engagement. You cannot be dialed in at an appropriate yardage setting with the Mil Dot system. The Mil Dot system should be used for what it was designed for which is range finding.

Range Estimating With The Mil. Dot Reticle

With practice, the Mil Dot system is simple to use. Dots are spaced in one mil (milliradian) increments on the crosshair. Using the mil formula, the shooter can create a table based on the known size of the object targeted. Just look through the scope, bracket the object between dots, and refer to the table for an estimated distance to the target.”

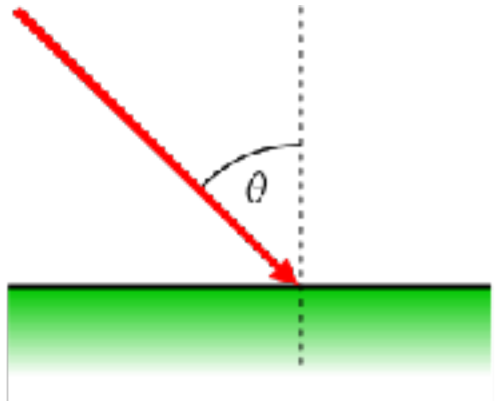
Elements that affect the use, (as described above) of the Milliradian (Mil-Dot) Reticle

According to the Scope manufacture as noted above, we now have a brief description of the mil-dot reticle and how to use it, both for ranging the distance to target and in substitution of dialing, the proper hold-over value in lieu of utilizing the scope’s turrets. However, due to certain laws of physics such as the law of refraction, known more specifically as “Snell’s law,” the angle of incidence, and in addition, ranging targets, the mil-dot reticle is confined to a narrow spectrum of use and is seriously handicapped when utilized in the mountainous areas as it is very possible if not a certainty that the Shooter will be aiming up or down on steep angles.

When utilizing the mil-dot hold over technique when aiming up or down on an angle, and at distances approximately at or beyond 300 meters, in lieu of adjusting the scope turrets, the bullets point of impact will be noticeably high.

When utilizing the ranging capability of the mil-dot reticle it works well when ranging targets that are not on an angle. Not so when ranging a target that is above or below you. This is because the target appears elongated. What may very well be one yard or meter in height, will appear to be taller when ranging a target that is on an angle.

Optics



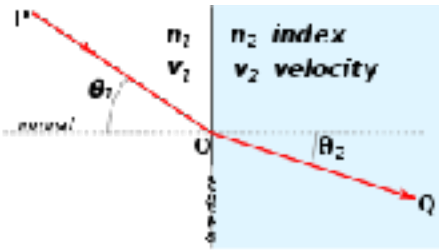
In geometric optics, the angle of incidence is the angle between a ray incident on a surface and the line perpendicular to the surface at the point of incidence, called the normal. The ray can be formed by any wave: optical, acoustic, microwave, X-ray and so on. In the figure above, the red line representing a ray makes an angle θ with the normal (dotted line). The angle of incidence at which light is first totally internally reflected is known as the critical angle. The angle of reflection and angle of refraction are other angles related to beams.

Refraction is the change in direction of a wave due to a change in its medium. It is essentially a surface phenomenon. The phenomenon is mainly in governance to the law of conservation of energy and momentum. Due to change of medium, the phase velocity of the wave is changed but its frequency remains constant. This is most commonly observed when a wave passes from one medium to another at any angle other than 90° or 0°. Refraction of light is the most commonly observed phenomenon, but any type of wave can refract when it interacts with a medium, for example when sound waves pass from one medium into another or when water waves move into water of a different depth. Refraction is described by “Snell’s law,” which states that for a given pair of media and a wave with a single frequency, the ratio of the sines of the angle of incidence θ_1 and angle of refraction θ_2 is equivalent to the ratio of phase velocities (v_1 / v_2) in the two media, or equivalently, to the opposite ratio of the indices of refraction (n_2 / n_1):

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}.$$

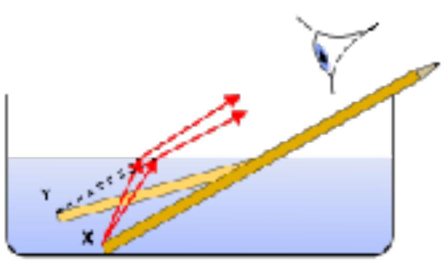
In general, the incident wave is partially refracted and partially reflected; the details of this behavior are described by the Fresnel equations.

Explanation



Refraction of light at the interface between two media of different refractive indices, with $n_2 > n_1$. Since the phase velocity is lower in the second medium ($v_2 < v_1$), the angle of refraction θ_2 is less than the angle of incidence θ_1; that is, the ray in the higher-index medium is closer to the normal. Consider the frequency of light in the air and how it then changes as it collides with the first objective lens of the rifle scope, and then ten times more for each additional lens.

In optics, refraction is a phenomenon that often occurs when waves travel from a medium with a given refractive index to a medium with another at an oblique angle. At the boundary between the media, the wave's phase velocity is altered, usually causing a change in direction. Its wavelength increases or decreases but its frequency remains constant. For example, a light ray will refract as it enters and leaves glass, assuming there is a change in refractive index. A ray traveling along the normal (perpendicular to the boundary) will change speed, but not direction. Refraction still occurs in this case. Understanding of this concept led to the invention of lenses and the refracting telescope.



An object (in this case a pencil) part immersed in water looks bent due to refraction: the light waves from X change direction and so seem to originate at Y. This explains the reason for the “wandering zero” throughout the day. When a shooter is shooting “flat,” not on an incline or decline, having zeroed their weapon / scope with the sun at its highest position above their head, and then clouds overcast the sky, the point of impact will be lower by about ¼ moa. If the position of the sun changes to the shooter’s nine o’clock position, the bullets point of impact will be to their right, approximately ¼ moa.

Refraction can be seen when looking into a bowl of water. Air has a refractive index of about 1.0003, and water has a refractive index of about 1.33. If a person looks at a straight object, such as a pencil or straw, which is placed at a slant, partially in the water, the object appears to bend at the water's surface. This is due to the bending of light rays as they move from the water to the air. Once the rays reach the eye, the eye traces them back as straight lines (lines of sight). The lines of sight (shown as dashed lines) intersect at a higher position than where the actual rays originated. This causes the pencil to appear higher and the water to appear shallower than it really is. The depth that the water appears to be when viewed from above is known as the ***apparent depth***. This is an important consideration for spear fishing from the surface because it will make the target fish appear to be in a different place, and the fisherman must aim lower to catch the fish.

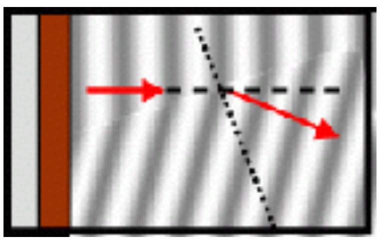


Diagram of refraction of water waves.

The diagram above shows an example of refraction in water waves. Ripples travel from the left and pass over a shallower region inclined at an angle to the wavefront. The waves travel slower in the more shallow water, so the wavelength decreases and the wave bends at the boundary. The dotted line represents the normal to the boundary. The dashed line represents the original direction of the waves. This phenomenon explains why waves on a shoreline tend to strike the shore close to a perpendicular angle. As the waves travel from deep water into shallower water near the shore, they are refracted from their original direction of travel to an angle more normal to the shoreline. Refraction is also responsible for rainbows and for the splitting of white light into a rainbow-spectrum as it passes through a glass prism. Glass has a higher refractive index than air. When a beam of white light passes from air into a material having an index of refraction that varies with frequency, a phenomenon known as dispersion occurs, in which different colored components of the white light are refracted at different angles, i.e., they bend by different amounts at the interface, so that they become separated. The different colors correspond to different frequencies.

While refraction allows for phenomena such as rainbows, it may also produce peculiar optical phenomena, such as mirages and Fata Morgana. These are caused by the change of the refractive index of air with temperature. Note; “Superior Mirage.”

Recently some metamaterials have been created which have a negative refractive index. With metamaterials, we can also obtain total refraction phenomena when the wave impedances of the two media are matched. There is then no reflected wave.

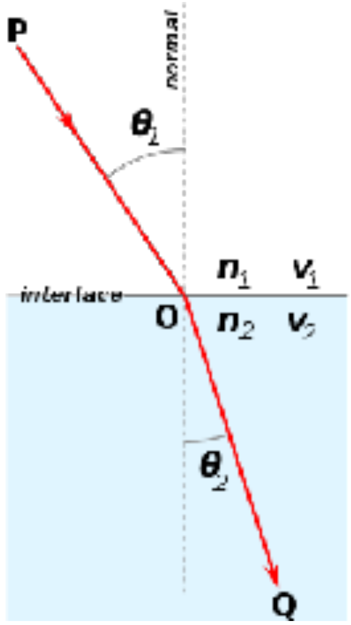
Also, since refraction can make objects appear closer than they are, it is responsible for allowing water to magnify objects. First, as light is entering a drop of water, it slows down. If the water's surface is not flat, then the light will be bent into a new path. This round shape will bend the light outwards and as it spreads out, the image you see gets larger.

A useful analogy in explaining the refraction of light would be to imagine a marching band as they march at an oblique angle from pavement (a fast medium) into mud (a slower medium). The marchers on the side that runs into the mud first will slow down first. This causes the whole band to pivot slightly toward the normal (make a smaller angle from the normal)

<u>The foundational principle of optical refraction and how light is affected by the collision of an introduced solid such as glass, is further exemplified by the explanation of Snell's law, and explains the reason why a bullet will impact high above the target when utilizing the mil-dot hold over technique when aiming up or down on an angle.</u>

Snell's Law

Snell's law (also known as the **Snell–Descartes law** and the **law of refraction**) is a formula used to describe the relationship between the angles of incidence and refraction, when referring to light or other waves passing through a boundary between two different isotropic media, such as water and glass.



Refraction of light at the interface between two media of different refractive indices, with $n_2 > n_1$. Since the velocity is lower in the second medium ($v_2 < v_1$), the angle of refraction θ_2 is less than the angle of incidence θ_1; that is, the ray in the higher-index medium is closer to the normal.

In optics, the law is used in ray tracing to compute the angles of incidence or refraction, and in experimental optics and gemology to find the refractive index of a material. The law is also satisfied in metamaterials, which allow light to be bent "backward" at a negative angle of refraction (negative refractive index).

Although named after Dutch astronomer Willebrord Snellius (1580–1626), the law was first accurately described by the Arab scientist Ibn Sahl at Baghdad court, when in 984 he used the law to derive lens shapes that focus light with no geometric aberrations in the manuscript ***On Burning Mirrors and Lenses*** (984).

Snell's law states that the ratio of the sines of the angles of incidence and refraction is equivalent to the ratio of phase velocities in the two media, or equivalent to the opposite ratio of the indices of refraction:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

with each θ as the angle measured from the normal, v as the velocity of light in the respective medium (SI units are meters per second, or m/s) and n as the refractive index (which is unitless) of the respective medium.

The law follows from Fermat's principle of least time, which in turn follows from the propagation of light as waves.

Explanation

Snell's law is used to determine the direction of light rays through refractive media with varying indices of refraction. The indices of refraction of the media, labeled n_1, n_2 and so on, are used to represent the factor by which a light ray's speed decreases when traveling through a refractive medium, such as glass or water, as opposed to its velocity in a vacuum.

As light passes the border between media, depending upon the relative refractive indices of the two media, the light will either be refracted to a lesser angle, or a greater one. These angles are measured with respect to the ***normal line***, represented perpendicular to the boundary. In the case of light traveling from air into water, light would be refracted towards the normal line, because the light is slowed down in water; light traveling from water to air would be refracted away from the normal line.

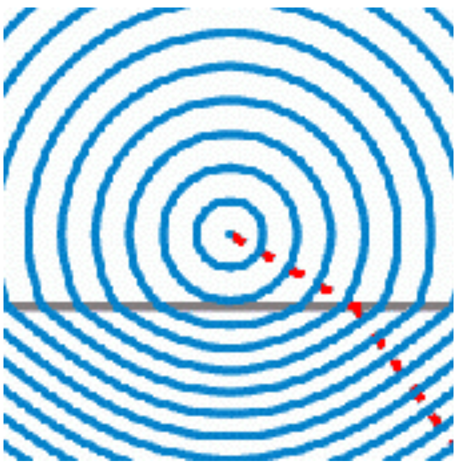
Refraction between two surfaces is also referred to as ***reversible*** because if all conditions were identical, the angles would be the same for light propagating in the opposite direction.

Snell's law is generally true only for isotropic or specular media (such as **glass**). In anisotropic media such as some crystals, birefringence may split the refracted ray into two rays, the ***ordinary*** or *o*-ray, which follows Snell's law, and the other ***extraordinary*** or *e*-ray which may not be co-planar with the incident ray; hence, the high point of impact of the bullet when aiming up or down on an angle utilizing the mil-dot hold over technique.

When the light or other wave involved is monochromatic, that is, of a single frequency, Snell's law can also be expressed in terms of a ratio of wavelengths in the two media, λ_1 and λ_2:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

Derivations and formula



Wavefronts from a point source in the context of Snell's law

The region below the grey line has a higher index of refraction, and proportionally lower speed of light, than the region above it.

Snell's law may be derived from Fermat's principle, which states that the light travels the path, which takes the least time. By taking the derivative of the optical path length, the stationary point is found giving the path taken by the light (though it should be noted that the result does not show light taking the least time path, but rather one that is stationary with respect to small variations as there are cases where light actually takes the greatest time path, as in a spherical mirror). In a classic analogy, the area of lower refractive index is replaced by a beach, the area of higher refractive index by the sea, and the fastest way for a rescuer on the beach to get to a drowning person in the sea is to run along a path that follows Snell's law.

Alternatively, Snell's law can be derived using interference of all possible paths of light wave from source to observer—it results in destructive interference everywhere except extrema of phase (where interference is constructive)—which become actual paths.

Another way to derive Snell’s Law involves an application of the general boundary conditions of Maxwell equations for electromagnetic radiation.

Yet another way to derive Snell's law is based on translation symmetry considerations. For example, a homogeneous surface perpendicular to the z direction cannot change the transverse momentum. Since the propagation vector $\vec{k}$ is proportional to the photon's momentum, the transverse propagation direction $(k_x, k_y, 0)$ must remain the same in both regions. Assuming without loss of generality a plane of incidence in the z, x plane $k_{x\mathrm{Region}_1}$ $k_{x\mathrm{Region}_2}$. Using the well known dependence of the wave number on the refractive index of the medium, we derive Snell's law immediately.

$$
\begin{aligned}
&k_{x\mathrm{Region}_1} \quad k_{x\mathrm{Region}_2} \\
&n_1 k_0 \sin\theta_1 \quad n_2 k_0 \sin\theta_2 \\
&n_1 \sin\theta_1 \quad n_2 \sin\theta_2
\end{aligned}
$$

where $k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega}{c}$ is the wave number in vacuum. Note that no surface is truly homogeneous, in the least at the atomic scale. Yet full translational symmetry is an excellent approximation whenever the region is homogeneous on the scale of the light wavelength.

Vector form

Given a normalized light vector **l** (pointing from the light source toward the surface) and a normalized plane normal vector **n**, one can work out the normalized reflected and refracted rays:

$\cos\theta_1 = \mathbf{n} \cdot (-\mathbf{l})$

$$
\begin{aligned}
\cos\theta_2 &= \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \left(1 - (\cos\theta_1)^2\right)} \\
\mathbf{v}_{\mathrm{reflect}} &= \mathbf{l} - (2\cos\theta_1)\,\mathbf{n} \\
\mathbf{v}_{\mathrm{refract}} &= \left(\frac{n_1}{n_2}\right)\mathbf{l} + \left(\frac{n_1}{n_2}\cos\theta_1 - \cos\theta_2\right)\mathbf{n}
\end{aligned}
$$

Note: $\cos\theta_1$ must be positive. Otherwise, use

$$\mathbf{v}_{\mathrm{refract}} = \left(\frac{n_1}{n_2}\right)\mathbf{l} - \left(\frac{n_1}{n_2}\cos\theta_1 - \cos\theta_2\right)\mathbf{n}.$$

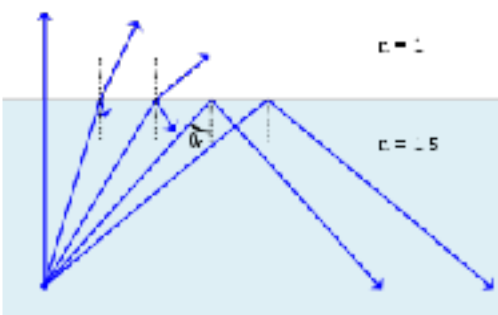
Example:

$$\mathbf{l} = \{0.707107, -0.707107\},\ \mathbf{n} = \{0, 1\},\ \frac{n_1}{n_2} = 0.9$$
$$\cos\theta_1 \quad 0.707107,\ \cos\theta_2 \quad 0.771362$$
$$\mathbf{v}_{\mathrm{reflect}} = \{0.707107, 0.707107\},\ \mathbf{v}_{\mathrm{refract}} = \{0.636396, -0.771362\}$$

The cosines may be recycled and used in the Fresnel equations for working out the intensity of the resulting rays.

Total internal reflection is indicated by a negative radicand in the equation for $\cos\theta_2$. In this case, an evanescent wave is produced, which rapidly decays from the surface into the second medium. Conservation of energy is maintained by the circulation of energy across the boundary, averaging to zero net energy transmission.

Total internal reflection and critical angle



Demonstration of no refraction at angles greater than the critical angle.

When light travels from a medium with a higher refractive index to one with a lower refractive index, Snell's law seems to require in some cases (whenever the angle of incidence is large enough) that the sine of the angle of refraction be greater than one. This of course is impossible, and the light in such cases is completely reflected by the boundary, a phenomenon known as total internal reflection. The largest possible angle of incidence, which still results in a refracted ray is called the **critical angle**; in this case the refracted ray travels along the boundary between the two media.

For example, consider a ray of light moving from water to air with an angle of incidence of 50°. The refractive indices of water and air are approximately 1.333 and 1, respectively, so Snell's law gives us the relation

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1.333}{1}\cdot\sin(50^\circ) = 1.333\cdot 0.766 = 1.021,$$

which is impossible to satisfy. The critical angle θ_{crit} is the value of θ_1 for which θ_2 equals 90°:

$$\theta_{crit} = \arcsin\left(\frac{n_2}{n_1}\sin\theta_2\right) = \arcsin\frac{n_2}{n_1} = 48.6^\circ.$$

Dispersion

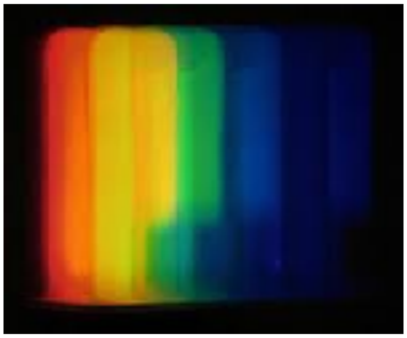
In many wave-propagation media, wave velocity changes with frequency or wavelength of the waves; this is true of light propagation in most transparent substances other than a vacuum. These media are called dispersive. The result is that the angles determined by Snell's law also depend on frequency or wavelength, so that a ray of mixed wavelengths, such as white light, will spread or disperse. Such dispersion of light in glass or water underlies the origin of rainbows and other optical phenomena, in which different wavelengths appear as different colors.

In optical instruments, dispersion leads to chromatic aberration; a color-dependent blurring that sometimes is the resolution-limiting effect. This was especially true in refracting telescopes, before the invention of achromatic objective lenses.

Dispersion (optics)

Dispersion of waves in optics

In a prism, material dispersion (a wavelength-dependent refractive index) causes different colors to refract at different angles, splitting white light into a rainbow.



A compact fluorescent lamp seen through an Amici prism

In optics, **dispersion** is the phenomenon in which the phase velocity of a wave depends on its frequency, or alternatively when the group velocity depends on the frequency. Media having such a property are termed ***dispersive media***. Dispersion is sometimes called ***chromatic*** **dispersion** to emphasize its wavelength-dependent nature, or **group-velocity dispersion** (**GVD**) to emphasize the role of the group velocity. Dispersion is most often described for light waves, but it may occur for any kind of wave that interacts with a medium or passes through an inhomogeneous geometry (e.g., a waveguide), such as sound waves.

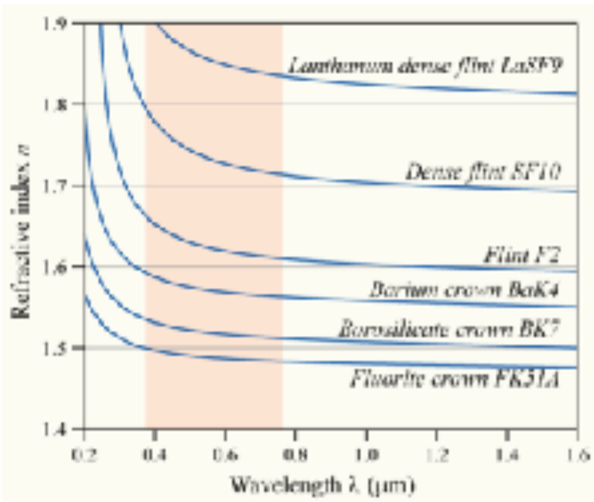
Examples of dispersion

The most familiar example of dispersion is probably a rainbow, in which dispersion causes the spatial separation of a white light into components of different wavelengths (different colors). However, dispersion also has an effect in many other circumstances: for example, GVD causes pulses to spread in optical fibers, degrading signals over long distances; also, a cancellation between group-velocity dispersion and nonlinear effects leads to soliton waves.

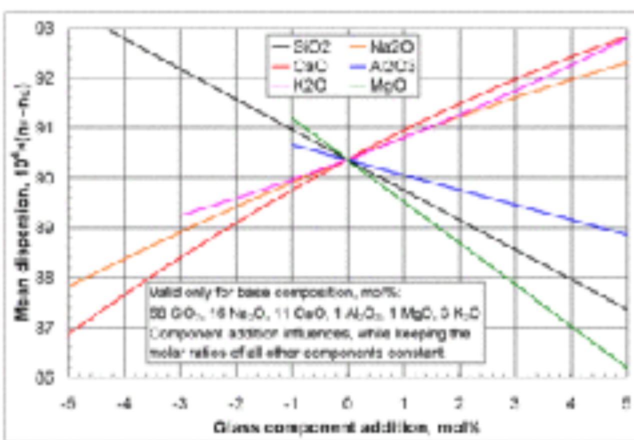
Sources of dispersion

There are generally two sources of dispersion: material dispersion and waveguide dispersion. **Material dispersion** comes from a frequency-dependent response of a material to waves. For example, material dispersion leads to undesired chromatic aberration in a lens or the separation of colors in a prism. **Waveguide dispersion** occurs when the speed of a wave in a waveguide (such as an optical fiber) depends on its frequency for geometric reasons, independent of any frequency dependence of the materials from which it is constructed. More generally, "waveguide" dispersion can occur for waves propagating through any inhomogeneous structure (e.g., a photonic crystal), whether or not the waves are confined to some region. In general, *both* types of dispersion may be present, although they are not strictly additive. Their combination leads to signal degradation in optical fibers for telecommunications, because the varying delay in arrival time between different components of a signal "smears out" the signal in time.

Material dispersion in optics



The variation of refractive index vs. vacuum wavelength for various glasses. The wavelengths of visible light are shaded in red.



Influences of selected glass component additions on the mean dispersion of a specific base glass (n_F valid for λ = 486 nm (blue), n_C valid for λ = 656 nm (red))

Material dispersion can be a desirable or undesirable effect in optical applications. The dispersion of light by glass prisms is used to construct spectrometers and spectroradiometers. Holographic gratings are also used, as they allow more accurate discrimination of wavelengths. However, in lenses, dispersion causes chromatic aberration, an undesired effect that may degrade images in microscopes, telescopes and photographic objectives.

The ***phase velocity***, ***v***, of a wave in a given uniform medium is given by

$$v = \frac{c}{n}$$

where *c* is the speed of light in a vacuum and *n* is the refractive index of the medium.

In general, the refractive index is some function of the frequency *f* of the light, thus $n = n(f)$, or alternatively, with respect to the wave's wavelength $n = n(\lambda)$. The wavelength dependence of a material's refractive index is usually quantified by its Abbe number or its coefficients in an empirical formula such as the Cauchy or Sellmeier equations.

Because of the Kramers–Kronig relations, the wavelength dependence of the real part of the refractive index is related to the material absorption, described by the imaginary part of the refractive index (also called the extinction coefficient). In particular, for non-magnetic materials ($\mu = \mu_0$), the susceptibility χ that appears in the Kramers–Kronig relations is the electric susceptibility χ_e $n^2 - 1$.

The most commonly seen consequence of dispersion in optics is the separation of white light into a color spectrum by a prism. From Snell's law it can be seen that the angle of refraction of light in a prism depends on the refractive index of the prism material. Since that refractive index varies with wavelength, it follows that the angle that the light is refracted by will also vary with wavelength, causing an angular separation of the colors known as ***angular dispersion*****.**

For visible light, refraction indices *n* of most transparent materials (e.g., air, glasses) decrease with increasing wavelength λ:

$$1 < n(\lambda_{\mathrm{red}}) < n(\lambda_{\mathrm{yellow}}) < n(\lambda_{\mathrm{blue}}) \ ,$$

or alternatively:

$$\frac{dn}{d\lambda} < 0.$$

In this case, the medium is said to have ***normal dispersion***. Whereas, if the index increases with increasing wavelength (which is typically the case for X-rays), the medium is said to have ***anomalous dispersion*****.**

At the interface of such a material with air or vacuum (index of ~1), Snell's law predicts that light incident at an angle θ to the normal will be refracted at an angle $\arcsin(\sin(\theta)/n)$.

Thus, blue light, with a higher refractive index, will be bent more strongly than red light, resulting in the well-known rainbow pattern.

Group and phase velocity

Another consequence of dispersion manifests itself as a temporal effect. The formula ***v* = *c* / *n*** calculates the ***phase velocity*** of a wave; this is the velocity at which the ***phase*** of any one frequency component of the wave will propagate. This is not the same as the ***group velocity*** of the wave, that is the rate at which changes in amplitude (known as the ***envelope*** of the wave) will propagate. For a homogeneous medium, the group velocity $\boldsymbol{v_g}$ is related to the phase velocity by (here λ is the wavelength in vacuum, not in the medium):

$$v_g = c\left(n - \lambda\frac{dn}{d\lambda}\right)^{-1}.$$

The group velocity v_g is often thought of as the velocity at which energy or information is conveyed along the wave. In most cases this is true, and the group velocity can be thought of as the ***signal velocity*** of the waveform. In some unusual circumstances, called cases of anomalous dispersion, the rate of change of the index of refraction with respect to the wavelength changes sign, in which case it is possible for the group velocity to exceed the speed of light ($\boldsymbol{v_g > c}$). Anomalous dispersion occurs, for instance, where the wavelength of the light is close to an absorption resonance of the medium. When the dispersion is anomalous, however, group velocity is no longer an indicator of signal velocity. Instead, a signal travels at the speed of the wavefront, which is *c* irrespective of the index of refraction. Recently, it has become possible to create gases in which the group velocity is not only larger than the speed of light, but even negative. In these cases, a pulse can appear to exit a medium before it enters. Even in these cases, however, a signal travels at, or less than, the speed of light, as demonstrated by Stenner, et al.

The group velocity itself is usually a function of the wave's frequency. This results in **group velocity dispersion** (GVD), which causes a short pulse of light to spread in time as a result of different frequency components of the pulse travelling at different velocities. GVD is often quantified as the ***group delay dispersion parameter*** (again, this formula is for a uniform medium only): (note: going from bright, unobstructed sunlight rays to cloud cover changes the frequency of the light as well as the group velocity dispersion, adding to a multiplied shift as well as variation in refractive properties).

$$D \quad -\frac{\lambda}{c}\,\frac{d^2n}{d\lambda^2}.$$

If *D* is less than zero, the medium is said to have ***positive dispersion*****.** If *D* is greater than zero, the medium has ***negative dispersion***. If a light pulse is propagated through a normally dispersive medium, the result is the higher frequency components travel slower than the lower frequency components. The pulse therefore becomes ***positively chirped*****,** or ***up-chirped***, increasing in frequency with time. Conversely, if a pulse travels through an anomalously dispersive medium, high frequency components travel faster than the lower ones, and the pulse becomes ***negatively chirped*****,** or ***down-chirped*****,** decreasing in frequency with time.

The result of GVD, whether negative or positive, is ultimately temporal spreading of the pulse. This makes dispersion management extremely important in optical communications systems based on optical fiber, since if dispersion is too high, a group of pulses representing a bit-stream will spread in time and merge, rendering the bit-stream unintelligible. This limits the length of fiber that a signal can be sent down without regeneration. One possible answer to this problem is to send signals down the optical fibre at a wavelength where the GVD is zero (e.g., around 1.3–1.5 μm in silica fibres), so pulses at this wavelength suffer minimal spreading from dispersion—in practice, however, this approach causes more problems than it solves because zero GVD unacceptably amplifies other nonlinear effects (such as four wave mixing). Another possible option is to use soliton pulses in the regime of anomalous dispersion, a form of optical pulse which uses a nonlinear optical effect to self-maintain its shape—solitons have the practical problem, however, that they require a certain power level to be maintained in the pulse for the nonlinear effect to be of the correct strength. Instead, the solution that is currently used in practice is to perform dispersion compensation, typically by matching the fiber with another fiber of opposite-sign dispersion so that the dispersion effects cancel; such compensation is ultimately limited by nonlinear effects such as self-phase modulation, which interact with dispersion to make it very difficult to undo.

Dispersion control is also important in lasers that produce short pulses. The overall dispersion of the optical resonator is a major factor in determining the duration of the pulses emitted by the laser. A pair of prisms can be arranged to produce net negative dispersion, which can be used to balance the usually positive dispersion of the laser medium. Diffraction gratings can also be used to produce dispersive effects; these are often used in high-power laser amplifier systems. Recently, an alternative to prisms and gratings has been developed: chirped mirrors. These dielectric mirrors are coated so that different wavelengths have different penetration lengths, and therefore different group delays. The coating layers can be tailored to achieve a net negative dispersion.

Dispersion in waveguides

Optical fibers, which are used in telecommunications, are among the most abundant types of waveguides. Dispersion in these fibers is one of the limiting factors that determine how much data can be transported on a single fiber.

The transverse modes for waves confined laterally within a waveguide generally have different speeds (and field patterns) depending upon their frequency (that is, on the relative size of the wave, the wavelength) compared to the size of the waveguide.

In general, for a waveguide mode with an angular frequency ω(β) at a propagation constant β (so that the electromagnetic fields in the propagation direction *(z)* oscillate proportional to $e^{i(\beta z-\omega t)}$), the group-velocity dispersion parameter *D* is defined as:

$$D = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2} = \frac{2\pi c}{v_g^2\lambda^2}\frac{dv_g}{d\omega}$$

where $\lambda = 2\pi c/\omega$ is the vacuum wavelength and $v_g = d\omega/d\beta$ is the group velocity. This formula generalizes the one in the previous section for homogeneous media, and includes both waveguide dispersion and material dispersion. The reason for defining the dispersion in this way is that |*D*| is the (asymptotic) temporal pulse spreading Δt per unit bandwidth $\Delta\lambda$ per unit distance travelled, commonly reported in ps / nm km for optical fibers.

A similar effect due to a somewhat different phenomenon is modal dispersion, caused by a waveguide having multiple modes at a given frequency, each with a different speed. A special case of this is polarization mode dispersion (PMD), which comes from a superposition of two modes that travel at different speeds due to random imperfections that break the symmetry of the waveguide.

Higher-order dispersion over broad bandwidths

When a broad range of frequencies (a broad bandwidth) is present in a single wavepacket, such as in an ultrashort pulse or a chirped pulse or other forms of spread spectrum transmission, it may not be accurate to approximate the dispersion by a constant over the entire bandwidth, and more complex calculations are required to compute effects such as pulse spreading.

In particular, the dispersion parameter *D* defined above is obtained from only one derivative of the group velocity. Higher derivatives are known as *higher-order dispersion*.[7]

These terms are simply a Taylor series expansion of the dispersion relation $\beta(\omega)$ of the medium or waveguide around some particular frequency. Their effects can be computed via numerical evaluation of Fourier transforms of the waveform, via integration of higher-order slowly varying envelope approximations, by a split-step method (which can use the exact dispersion relation rather than a Taylor series), or by direct simulation of the full Maxwell's equations rather than an approximate envelope equation.

How does this effect the use of holding over with a mil-dot or milliradian reticle when aiming up or down on an angle?

The over all effect of the previously mentioned laws cause the light rays to bend downwards when holding over on the reticle's milliradian dots or marks while aiming up or down on an angle. This in turn causes the point of impact to be high. This last year I had the opportunity to train up SOF Sniper's at the Mountain Shooting Center. While at an altitude of 9,400' ASL, I had them correct for gravity and then dial in their dope while aiming downwards on an angle of approximately 25 degrees. The distance to target was approximately 550 meters and the target size was 1 meter tall, (39"). Everyone of the Sniper's hit their targets. For the next evolution, I had them return their turrets to their zeroes and then hold over on their mil-dots while aiming at the same targets. Every Sniper missed their target with the bullet going over the very top of the target by approximately 12". This equated to a high point of impact miss, above center of 30".

Some of the other contributing factors are and could be as follows:

1. Picatinny rail with 20 moa;
2. Curvature of the eleven lenses within the scope;
3. "Down-Chirping," through the eleven lenses;
4. 600 meter Zero;
5. Wind;
6. Superior Mirage.

Add the additional 20 moa down-slope of the scope and the law of refraction is magnified.

The refracted light and secondary light transmitted through the eleven lenses of the scope further magnify the negative properties of the law of refraction, as well as "down-chirping."

The adjusted "hold" on the reticle for the 550 meter distance to target, or roughly 3 mils, pulls the eye away from the scope's optical center and into the curvature of the lenses, further magnifying the effects of the law of refraction.

Winds cause some anomalies such as “Superior Mirage” to be somewhat dispersed. However, without a breeze, the Superior Mirage will also bend light rays downwards; causing the shooter to aim above the true location of the target as well. In essence, when utilizing hold overs, the image of the target is being lifted up as well.

If the effects of gravity are not taken into account and corrected, the point of impact will also be high.

Superior Mirage

Superior mirage of a distant land

A **superior mirage** occurs when the air below the line of sight is colder than that above. This is called a temperature inversion, since it does not represent the normal equilibrium temperature gradient of the atmosphere. In this case the light rays are bent down, the image appears above the true object, hence the name ***superior.*** They are in general less common than inferior mirages, but when they do occur they tend to be more stable, as cold air has no tendency to move up and warm air no tendency to move down.

Superior mirages are most common in polar regions, especially over large sheets of ice with a uniform low temperature. They also occur at more moderate latitudes, however, although in that case they are weaker and not so smooth. For example a distant shoreline may be made *towering*, looking higher (and thus perhaps closer) than it is in reality, but because of the turbulences there seem to be dancing spikes, towers and so forth. Superior mirage can also occur in the Great Plains when it is hot and the shooter is shooting through pockets of cool air, such as over marshes, streams, ponds wadi's. In thr mountains, this can also occur when shooting up at heated ridgelines through the cooler air as it is drafted up and through the canyons. This type of mirage is also called the "Fata Morgana or in Icelandic, *halgerndingar*.

Superior images can be straight up or upside down, depending on the distance of the true object and the temperature gradient. Often the image appears as a distorted mixture of up and down parts.

Superior mirage of the boats at the entrance of the harbor at Victoria, British Columbia, Canada

The image contains three frames. The main frame and the image on the right show superior **mirage** as it is seen across Monterey Bay from Santa Cruz. The image on the left shows the same place with no mirage. All images were photographed on different days. The diagram shows the light rays bending due to superior **mirage** conditions.

If the Earth were flat, superior images would not be as interesting. Light rays which bent down would soon hit the ground, and only close objects would be affected. Since the Earth is round, if the amount of downward bending is about equal to the curvature of the Earth, light rays can travel large distances, perhaps from beyond the horizon. This was observed for the first time in 1596, when a ship under the command of Willem Barents looking for the Northeast Passage got stuck in the ice at "Novaya Zemlya" and the crew had to endure the polar winter there. They saw their midwinter night ending with the rise of a distorted sun about 2 weeks earlier than expected. It was not until the 20th century that Europeans understood the reason: that the real sun had still been under their horizon, but its light rays followed the curvature of the Earth. This effect is often called a Novaya Zemlya mirage. For every 100 km the light rays can travel parallel to the Earth's surface, the sun will appear 1° higher on the horizon. The inversion layer must have just the right temperature gradient over the whole distance to make this possible. In the same way ships which are in reality so far away that they should not have been visible above the geometric horizon, may appear on the horizon, or even above the horizon as superior mirages. This may explain some stories about flying ships or coastal cities in the sky, as described by some polar explorers. These are examples of so called **Arctic mirages** or *hillingar* in Icelandic.

If the vertical temperature gradient is +11°C per 100 meters (reminder: positive means getting hotter when going up), horizontal light rays will just follow the curvature of the Earth, and the horizon will appear flat. If the gradient is less the rays are not bent enough, and get lost in space. That is the normal situation of a spherical, convex horizon. But if the gradient gets larger, say 18°C per 100 meters, the observer will see the horizon turned upwards, being concave, as if he were standing on the bottom of a saucer.

It is important to understand what mirage is all about for several reasons.

1. Mirage will tell you the direction, speed and personality or pattern of the wind, which in turn will help you make a first round reduction.
2. Mirage will tell you where your target really is.

While shooting in an area known as the Great Plains, the distance to target was 1050 yards. I went from center hits to missing high by fifteen feet in an instant. As I shot over the marsh and through the cool pockets of air, I was experiencing a ***Superior Mirage*** condition.

What does shooting in the plains have to do with shooting in mountainous high angle areas?

It helps tell the whole story of how the different types of mirage can have an affect your point of impact.

Where and when can a *Superior Mirage* occur in mountainous regions?
It can happen when shooting over bodies of water and over bodies of ice and snow on summer days and it can occur when shooting up or down on angles at longer distances. As an example, the rule of the sun (the heating of a mountain-side) will cause air flow rise in the direction of the mountain side. Aiming up at an 1100 meter target on a 15+ degree hold that crosses the air flow, will cause cool air to pass under the warmer air as it is drawn up the mountain face. This in turn will cause a point of impact shift and a higher strike.

How can you prevent a miss? <u>To determine a Superior Mirage,</u> place the power setting of your scope on its highest power and aim at your target. Next, look at your ACI and determine the angle or cosine that is being indicated. Next, turn the power of your scope all the way down and again, look at your ACI and determine the angle or cosine that is being indicated. If there is a change, and the ACI is indicating a lesser angle of incident, then there is a Superior mirage present. Shoot on the lower power and or with a wind. Even a slight wind will scatter a superior mirage. High power magnification always magnifies the anomalies that you may or may not see and mountain environments and high angle shooting are always challenging.

One more example of shooting in a Superior Mirage condition is as follows. I sometimes shoot with a friend who owns thirty-five acres of land. As such, we have set up a target at 386 meters / 419 yards. The lay of the land is as follows: The target is on a hillside, upwards on a ten degree angle. The shooting station is on a mound that gradually slopes downwards towards a stream bed at 229 meters / 250 yards, and then the ground aggressively slopes upwards to the target. The temperature at the shooting station is 87 degrees F. At the stream bed the temperature is 75 degrees F, with cool air slowly moving up through it. This creates a superior mirage condition. When the air is static at the shooting station, but moving through the stream bed, the shooter looks up at the target through the cooler air. When the round is sent down-range to the target, it strikes six inches high. With this scenario, the shooter will hold low six inches. However, when a slow moving 1.5 mph tail wind is present, it hits the hill-side, building pressure and stalls the cooler air. The shooter then holds center.